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BY

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Design and Operation of an Improved Counting Rate Meter

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The counting rate meter is an electronic amplifier and computing circuit whose output is a d.c. current or voltage proportional to the number of pulses fed into the circuit. The input pulses may be either uniformly spaced or distributed randomly in time, as in the most common use of the instrument as an amplifier and recorder for use with Geiger-Müller counters. The electronic design and operation is discussed for each of the circuit components: amplifiers, pulse equalizer, integrating circuit, degenerative vacuum-tube voltmeter, and the stabilized high voltage and low voltage supplies. The statistical interpretation of the counting rate meter output readings due to the randomly distributed pulses from radioactive sources requires a special statistical theory because an integrating and averaging circuit produces an exponential interdependence of successive observations on all preceding observations. Practical methods, with curves, are developed for determining the mean counting rate and the probable error of the mean rate directly from the output records.

1. INTRODUCTION

THE counting rate meter is an amplifier for use with Geiger-Müller counters, or any other random or periodic counting instruments. It includes a simple electrical computing circuit such that the output reads the average counting rate directly. The instrument has been improved[1] in a number of ways since it was first described.[2] Simplified forms of the basic circuits have been described for use as γ-ray dose meters.[3] It is the purpose of the present paper to discuss the important modifications[4] made several years ago, and tested over the past four years by the continuous use of some twenty instruments of the type described for physical, biological, and metallurgical research using radioactive isotopes. The statistical analysis[5] of counting rate meter data, especially the direct observation of the probable error of the average counting rate, is also discussed.

As in the original design, the unit consists of a high voltage supply and associated circuit for operating a Geiger-Müller counter, amplifiers, and a pulse equalizer for insuring that all pulses from the amplifier are the same shape and size.

[1] R. D. Evans and R. L. Alder, Rev. Sci. Inst. **10**, 332–336 (1939); R. D. Evans and R. Meagher, Rev. Sci. Inst. **10**, 339–344 (1939).
[2] N. S. Gingrich, R. D. Evans, and H. E. Edgerton, Rev. Sci. Inst. **7**, 450–456 (1936).
[3] L. F. Curtiss, Nat. Bur. Stand. J. Research **23**, 137–143 (1939); *ibid.* **23**, 479–484 (1939); L. F. Curtiss and B. W. Brown, *ibid.* **34**, 53–58 (1945).
[4] A. F. Kip and R. D. Evans, Phys. Rev. **59**, 920A (1941).
[5] L. I. Schiff and R. D. Evans, Rev. Sci. Inst. **7**, 456–462 (1936).

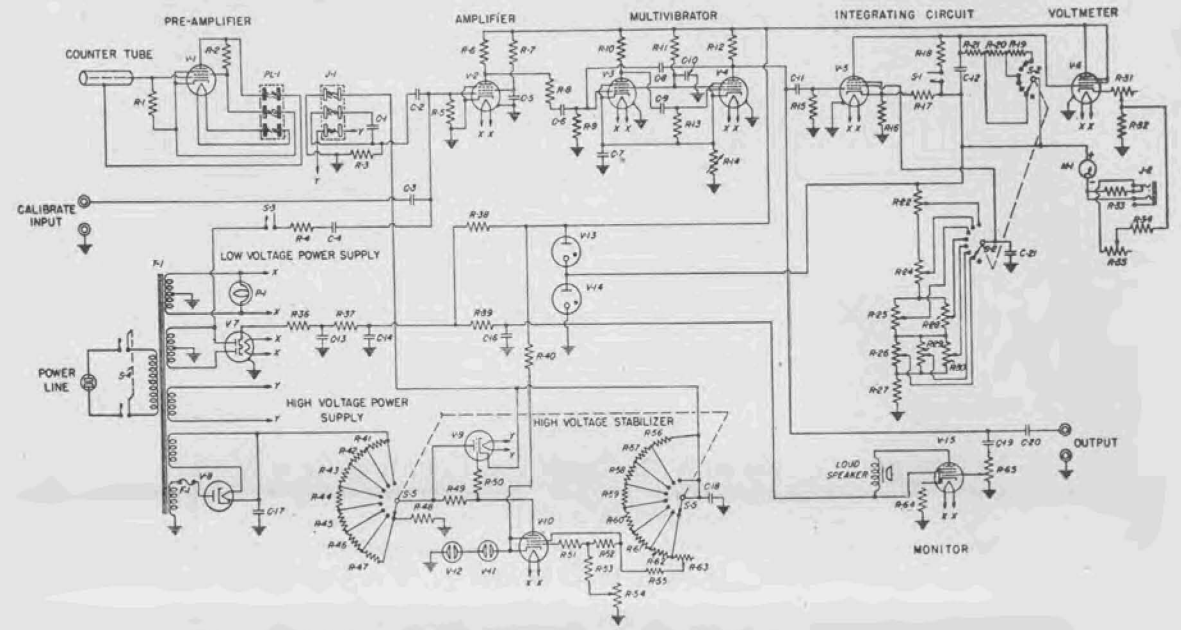

FIG. 1. Parts list, resistances in ohms.

V-1	6J7 or 6C6
V-2	6SJ7
V-3	6SJ7
V-4	6SJ7
V-5	6SJ7-GT/G
V-6	6AC7/1852
V-7	6X5
V-8	2X2/879
V-9	6C5
V-10	6C6
V-11, *V*-12	$\frac{1}{4}$ watt neon bulbs, bayonet base without internal resistors
V-13	VR105–30
V-14	VR105–30
V-15	6K6-GT/G
T-1	GR-365–428 or RCA No. 33,390
R-1	10 meg 1 watt
R-2	5 meg 1 watt
R-3	6 meg (2000 v)
R-4	1 meg 1 watt
R-5	0.5 meg $\frac{1}{4}$ watt
R-6	200,000 1 watt
R-7	300,000 1 watt
R-8	1 meg $\frac{1}{4}$ watt
R-9	100,000 $\frac{1}{2}$ watt
R-10	250,000 1 watt
R-11	300,000 $\frac{1}{2}$ watt
R-12	250,000 1 watt
R-13	100,000 $\frac{1}{4}$ watt
R-14	2000 1 watt variable
R-15	0.5 meg $\frac{1}{4}$ watt
R-16	3300 10 watt
R-17	30,000 1 watt
R-18	100,000 $\frac{1}{4}$ watt
R-19	10 meg $\frac{1}{4}$ watt
R-20	10 meg $\frac{1}{4}$ watt
R-21	10 meg $\frac{1}{4}$ watt
R-22	50,000 1 watt variable
R-23	0
R-24	25,000 1 watt variable
R-25	15,000 1 watt variable
R-26	15,000 1 watt variable
R-27	8500 1 watt
R-28	15,000 1 watt variable
R-29	15,000 1 watt variable
R-30	15,000 1 watt variable
R-31	300 1 watt variable
R-32	12,000 10 watt
R-33	1000 $\frac{1}{2}$ watt
R-34	1000 1 watt
R-35	2000 1 watt variable
R-36	150 $\frac{1}{2}$ watt variable
R-37	1000 10 watts variable
R-38	3500 10 watts variable
R-39	10,000 10 watts variable
R-40	300,000 2 watts variable
R-41	0.1 meg $\frac{1}{4}$ watt variable
R-42	0.2 meg $\frac{1}{4}$ watt variable
R-43	0.3 meg $\frac{1}{4}$ watt variable
R-44	0.3 meg $\frac{1}{4}$ watt variable
R-45	0.25 meg $\frac{1}{4}$ watt variable
R-46	0.25 meg $\frac{1}{4}$ watt variable
R-47	0.25 meg $\frac{1}{4}$ watt variable
R-48	2.25 meg $\frac{1}{4}$ watt variable
R-49	100 meg *IRC* metalized
R-50	10 meg 1 watt
R-51	1 meg $\frac{1}{4}$ watt
R-52	0.5 meg $\frac{1}{4}$ watt
R-53	0
R-54	1 meg 1 watt variable
R-55	1 meg $\frac{1}{4}$ watt variable
R-56	0.75 meg $\frac{1}{4}$ watt variable
R-57	0.75 meg $\frac{1}{4}$ watt variable
R-58	0.75 meg $\frac{1}{4}$ watt variable
R-59	0.75 meg $\frac{1}{4}$ watt
R-60	0.75 meg $\frac{1}{4}$ watt variable
R-61	0.75 meg $\frac{1}{4}$ watt variable
R-62	0.75 meg $\frac{1}{4}$ watt variable
R-63	1 meg linear, 1 watt variable
R-64	2000 10 watts
R-65	0.5 meg 1 watt variable
C-1	0.5 μf 1000 volts paper
C-2	50 $\mu\mu$f 3000 volts ceramic
C-3	0.01 μf 600 volts paper
C-4	0.002 μf mica
C-5	0.25 μf paper
C-6	5 $\mu\mu$f mica
C-7	0.5 μf paper 400 volts
C-8	0.0001 μf mica
C-9	0.0001 μf mica
C-10	0.25 μf paper
C-11	0.0002 μf mica
C-12	2 μf polystyrene
C-13	20 μf electrolytic
C-14	20 μf electrolytic
C-16	20 μf electrolytic
C-17	0.2 μf paper 2000 volt
C-18	0.25 μf paper 2000 volt
C-19	50 $\mu\mu$f mica
C-20	50 $\mu\mu$f mica
C-21	0.1 μf paper
F-1	$\frac{1}{32}$ ampere fuse

The equalized pulses are fed into a tank circuit consisting of a condenser and resistance. The voltage developed across this condenser is proportional to the average rate at which pulses come from the counter, and thus is proportional to the activity of the radioactive sample.

Changes have been made in the voltage supplies, all batteries have been eliminated, and the counting-rate range of the instrument has been increased. In the present design the voltage developed across the integrating condenser is measured by a degenerative vacuum-tube voltmeter whose output is read on a panel meter calibrated directly in terms of counts per minute. The vacuum-tube voltmeter circuit increases the available full scale current 50-fold, from 100 μa. to 5 ma, allowing the use of a commercial pen and ink recording milliammeter in place of the photographic technique required for recording with the earlier designs. The vacuum-tube voltmeter circuit has also made it possible to increase the resistance in the *RC* integrating circuit with a corresponding decrease in capacity from 100 to 2 μf. The wiring diagram is given in Fig. 1.

2. THE AMPLIFIERS

The Geiger-Müller counter tube and a Neher-Pickering[6] type preamplifier, modified to eliminate the screen-grid battery, are built into a single shielded container. Space requirements are small. The counter and preamplifier assembly is connected to the counting-rate meter proper by means of a shielded cable and a standard six-terminal plug, modified by using micarta brackets and shorter mounting screws so as to increase the leakage path, since this plug carries the high voltage to the counter.

A second amplifier stage has been introduced between the preamplifier and the pulse equalizer. This insures that a minimum sized pulse from even a very small helium-filled counter will be sufficient to actuate the pulse equalizer. It also reduces the pulse width to less than the relaxation time of the pulse equalizer[7] (*ca.* 6×10^{-5} sec., by Eq. (1.51a) of reference 7). Thus in no case will a single pulse allow the equalizer to give more than one pulse.

The preamplifier is coupled to the second amplifier stage through a small condenser (*C*-2) which has the full counter voltage (400–2000 volts) across it. Hence the condenser must be designed to withstand the high voltage without introducing spurious discharges because of leakage.

3. THE PULSE EQUALIZER

The equalizer is a conventional multivibrator, as described earlier for this circuit,[2] biased below cut-off. It is inserted in the circuit to insure that all the pulses from the counter are made uniform in size and shape before they reach the integrating circuit. Metal tubes (*V*-3, *V*-4) should be used, or adequate shielding employed if glass tubes must be substituted. The number and time distribution of the pulses from the multi-vibrator is determined by the pulses from the Geiger-Müller counter. Pentode tubes are used rather than triodes in order to obtain a maximum plate current and hence charge output per pulse, at the same time reducing the resolving time.

[6] H. V. Neher and W. H. Pickering, Phys. Rev. **53**, 316L (1938).

[7] E. R. Shenk, Electronics **17**, 138 (1944), see p. 350.

4. INTEGRATING CIRCUIT

The equalized pulses are of positive polarity and, when impressed on the grid of a tube (*V*-5), cause corresponding pulses of current to flow momentarily through a resistor (*R*-19, 20, 21) in the plate circuit of the tube, thus building up a d.c. voltage across a condenser (*C*-12) in parallel with the resistor. This voltage is proportional to the average pulse rate.[5] A d.c. vacuum-tube voltmeter across this integrating tank circuit measures this voltage and is calibrated to indicate directly the pulse-rate in counts per minute. The voltmeter deflection for a given counting rate is determined by the size and shape of the pulses from the equalizer, the operating characteristics of the tube supplying the tank circuit, the resistance in the tank circuit, and the voltmeter sensitivity. Once the control pulses have been equalized, therefore, permanence of the counting-rate calibration of the voltmeter depends on the generation of charging pulses of constant magnitude by the tube (*V*-5) and on the maintenance of constant sensitivity of the voltmeter circuit.

The amplitude of the charging pulses depends on the biasing control-grid voltage of the tube (*V*-5) and also very greatly on the screen voltage. Both voltages are obtained from voltage dividers across the VR 105–30 regulator tube (*V*-14). For maximum tube life *R*-16 should be adjusted to give 10.5 volts negative grid bias.

Variation of the screen voltage has proved the most satisfactory method of varying the counting rate range of the meter scale, the voltmeter sensitivity being held constant. To provide full scale output deflection for a wide range of counting rates, seven discrete values for the screen-grid voltage and appropriate values for the discharge resistance of the integrating circuit are provided by the counting-range selector switch (*S*-2). Using a source of pulses of known frequency, such as from the pulse generator described in Section 8, the screen-grid potentiometers (*R*-22 to *R*-30) are individually adjusted to give full-scale output for the counting rates shown in Table I. This pulse generator can be incorporated in the same chassis and can use the same power supply as the counting rate meter. The discharge resistance (*R*-19 to *R*-21) takes on

TABLE I. Counting rates for full scale (5 ma) output, and integrating circuit time constants RC, for the seven counting ranges.

Range	Maximum counting rate per minute	Time constant seconds
1	20,000	20
2	10,000	40
3	5000	40
4	2000	40
5	1000	60
6	500	60
7	200	60

three values (30, 20, or 10 megohms) as the ranges are changed to provide an adequate input to the voltmeter stage. With the tank condenser (C-12) fixed at 2 μf these changes in the discharge resistance also automatically change the time-constant of the integrating circuit from 60 seconds for low counting rates to 40 seconds for intermediate rates and 20 seconds for the highest counting rates. In Fig. 1, the top contact in both portions of S-2 is for the lowest counting rate, range 7.

The tank condenser has polystyrene dielectric to avoid polarization. Any good paper condenser may be used, but appreciable hysteresis effects caused by polarization will be found; that is, even if the charged condenser is shorted for some time, the charge is not completely dissipated; when the short is removed, a voltage will gradually appear across the condenser terminals. The special condensers used in the present design were wound with polystyrene tape instead of paper dielectric. They show no hysteresis effect, even when the resistance across them is made as high as 30 megohms. A push-button switch (S-1) is provided to permit momentarily shunting the tank circuit condenser with a 100,000-ohm resistor. This is very useful to accelerate the procedure between measurements, since the condenser can be partially discharged to the approximate value for the next measurement or it can be fully discharged. If the condenser used does not have polystyrene dielectric and exhibits hysteresis, the shunting resistor should be reduced to, say, 300 ohms, and the shunting switch should be closed for at least 30 seconds. In this case it is preferable to use an *on-off* switch rather than the push-button type for shunting the condenser. The tank condenser should be well protected against humidity.

If the tube used in the integrating circuit is replaced, the screen-grid voltages must be readjusted. The recalibration should be delayed until the tube has been aged in the circuit at least 24 hours.

The minimum value of the grid resistor R-15 is chosen to be sufficiently high to prevent the excessive loading of the multivibrator output circuit.

5. VACUUM-TUBE VOLTMETER

The vacuum-tube voltmeter employs cathode-circuit degeneration to stabilize the sensitivity. With this type of circuit the change in plate current, Δi, because of a change in the applied voltage, ΔE, is

$$\Delta i=(\Delta E/R)[1/(1+1/\mu+1/GR)], \qquad (1)$$

where μ is the amplification factor, and G the transconductance of the tube. R is the degenerative resistance. In Fig. 1, it consists of the resistive circuit from the cathode of the voltmeter tube (V-6) to ground, since this circuit is common to both the plate and the grid circuits. The voltmeter tube is a 6AC7-type operated as a triode (screen and suppressor grids connected to the plate). In addition to supplying sufficient output to operate a 5-ma recorder, the tube has a high amplification factor and high transconductance. The plate current change then becomes very nearly

$$\Delta i=\Delta E/R. \qquad (2)$$

As a result, the vacuum-tube voltmeter is linear and is quite independent of the tube characteristics.

Since the voltmeter must measure the voltage of the tank circuit, which is at a high potential above ground, the voltmeter and integrating circuits are combined in a configuration devised by Tuttle[8] in which the cathode degeneration network raises the cathode of the voltmeter tube above ground by the plate voltage of the integrating tube plus the desired control-grid bias, and the voltage of the regulator tube supplying the electrode voltages of the integrator tube also provides the bucking voltage for the meter circuit.

[8] W. N. Tuttle, U. S. Pat. 2,374,248, April 24, 1945.

These connections also balance out in the meter circuit the effect of residual fluctuations in the power supply, giving a particularly stable meter indication. Any small fluctuation in the voltage of the regulator tube (*V*-14) appears in series with the voltage of the integrating condenser in the grid circuit of the voltmeter tube. Because of the degenerative effect, it follows from (1) that this same voltage, reduced only in the ratio $1+1/\mu+1/GR$, appears between the cathode of the voltmeter tube and ground and in a sense to oppose the current through the meter resulting from the original disturbance across *V*-14. With the constants of the present circuit, fluctuations are reduced in a ratio of about 1:25.

Returning the meter to the regulator tube junction also provides a low resistance source of bucking voltage, and the total degenerative resistance becomes *R*-32 in parallel with the resistance of the meter branch of the circuit. With the present circuit the degenerative resistance can readily be made low enough to obtain the desired sensitivity and still maintain the cathode at the required potential. A good compromise between sensitivity and stability is a voltmeter sensitivity of about 16.5 volts on the tank condenser (*C*-12) for full-scale (5 ma) deflection of the output meter (*M*-1).

The small resistor (*R*-31) at the cathode is provided for adjusting the zero of the voltmeter, and is small enough to have a negligible effect on the calibration. The 6AC7 voltmeter tube should be replaced when it is no longer possible to adjust the voltmeter zero with *R*-31.

6. THE RECORDER

For continuous routine operation and for highest accuracy it is considered essential to have a method of continuously recording the reading of the counting rate meter. Photographic recording from a galvanometer may be used, but the use of a 5-ma commercial pen and ink recording milliammeter offers numerous advantages over the photographic method. In particular, the ability to observe the record at the time of running the sample is of utmost utility, and there is also a saving in time because of the elimination of photographic developing. The linearity of some commercially available recorders is excellent. A paper speed of 6 inches per hour has been found satisfactory for most observations.

The recorder is connected in series with the meter. In the present design it is inserted by a plug and jack arranged with a connecting cable. When the recorder is removed, the jack actuates a switch to introduce a resistance (*R*-33) that supplants the resistance of the recorder.

7. ACCESSORY CIRCUITS

The output of the pulse equalizer is made available for operating an external scaling circuit, if desired, through the jacks marked "output" on Fig. 1.

At the same point in the circuit, a monitoring amplifier which feeds a loudspeaker is inserted. The signal lead to the speaker should be shielded to prevent any coupling with the output circuit.

A signal input plug marked "calibrate input" is provided which feeds into the second amplifier. This is useful for connecting a variable frequency generator for calibrating the seven counting ranges. (See Section 8.)

A calibration switch (*S*-3) connects a signal from the line voltage to the second amplifier through a condenser. This is useful in testing the calibration of the apparatus at a pulse rate determined by the line frequency (3600 pulses per minute for a 60-cycle per second power line frequency). Usually, if this calibration is found to be correct, the other ranges are also satisfactory, provided they have been calibrated previously as described in Section 4.

8. ACCESSORY CALIBRATING SIGNAL GENERATOR

An accurate and convenient method of calibrating the instrument and periodically checking the calibration is by means of a multivibrator pulse generator synchronized at various ratios, of the 60-cycle line frequency.[9] The circuit shown in Fig. 1A has been employed in which a 60-cycle synchronizing signal is fed into the grids of the multivibrator. This signal locks the multivibrator on to frequencies of 200, 450, 900, 1800, 3600, 7200, and 14,400 pulses per minute, to which it is tuned by changing *R* and *C* in the circuit. In order to insure that the signal generator is

[9] F. E. Terman, *Radio Engineering* (McGraw-Hill Book Company, Inc., New York, 1932).

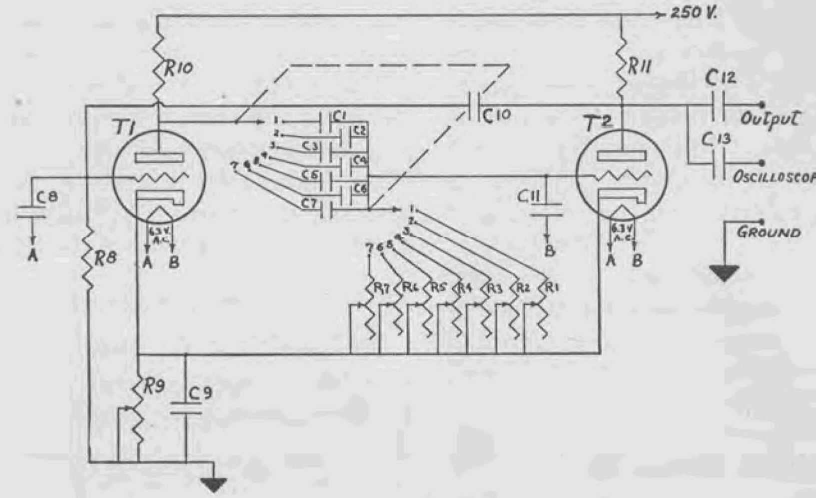

FIG. 1A. Parts list for pulse generator.

*R*1	500,000 ohm pot.	*R*6	500,000 ohm pot.	*C*1	.003 μf	*C*8	.003 μf
*R*2	500,000 ohm pot.	*R*7	500,000 ohm pot.	*C*2	.006 μf	*C*9	.5 μf
*R*3	500,000 ohm pot.	*R*8	125,000 ohms	*C*3	.01 μf	*C*10	.05 μf
*R*4	500,000 ohm pot.	*R*9	10,000 ohm pot.	*C*4	.02 μf	*C*11	.003 μf
*R*5	500,000 ohm pot.	*R*10	200,000 ohms	*C*5	.15 μf	*C*12	.00005 μf
		*R*11	200,000 ohms	*C*6	.40 μf	*C*13	.00001 μf
				*C*7	1.0 μf	*T*1 and *T*2	6SC7

locking onto the appropriate frequency, it is necessary to monitor the signal and compare it with a 60-cycle signal on an oscilloscope.

9. LOW VOLTAGE SUPPLY

The low voltage power supply is of standard design. The arrangement shown was chosen because of reasonable cost and small space requirements. The three 20-mf filter condensers (*C*-13, *C*-14, *C*-16) are in one small container. Two VR105–30 regulator tubes provide stabilized low voltage (210 volts) to all circuits except the monitor. The monitor supply should be isolated as shown to avoid possible intercoupling.

All vacuum tube heaters are supplied by the same transformer winding, except for the high voltage rectifier tube, which is on a separate winding, and the preamplifier tube and high voltage stabilizer tube (6C5), which share another separate winding.

10. HIGH VOLTAGE SUPPLY

Stabilized high voltage is provided which is constant to better than ±5 volts at 2000 volts for input voltages of 105 to 125. The internal resistance of the stabilizer is low, so that changing load, within the range of loads encountered, has a negligible effect on the voltage.

The high voltage stabilizer is of the degenerative feedback type, in which a signal voltage is taken off the output resistor (*R*-52 to *R*-63) and fed back through an amplifier (*V*-10) to a control tube (*V*-9). In this type of stabilizer the stabilization ratio depends on the over-all amplification between the output voltage and the control tube and on the constancy of the reference or bias voltage. The input voltage range over which stabilization is maintained depends on the working range of the vacuum tubes (especially the plate voltage of *V*-9) and the range over which the reference voltage (cathode bias of *V*-10) is constant.

For the values of resistors given, the high voltage transformer should give about 2400 volts after rectification. This will give over 2000 volts maximum stabilized output. The high voltage input filter condenser (*C*-17) can be as low as 0.15 mf, and can be three 0.5 μf, 1000-volt paper condensers in series, tapped into *R*-41 to *R*-48 to maintain voltage division. The 60-cycle ripple in the high voltage output depends somewhat on the filter condenser but is mainly suppressed by the stabilizer tubes. With 0.1 mf at *C*-17, the ripple is only of the order of 1 volt at all output voltages. The use of the 2X2 rectifier tube, which has a heater cathode, avoids the necessity of a time delay switch in the supply voltage line. When properly adjusted for maximum stabilization, the voltage drop in the 6C5 regulator tube (*V*-9) varies from 150 to 600 volts. The two-gang tap switch (*S*-5) provides stabilized voltages ranging from 400 to 2000 volts, in 200 volt steps. The potentiometer (*R*-63) used as a vernier control on this voltage should be linear to provide linear voltage control between voltage steps. Another potentiometer (*R*-54) is located behind the panel for adjusting the maximum voltage output of the stabilizer. With *R*-54 adjusted to provide a maximum of 2050 volts, the vernier *R*-63 provides enough variation to make the voltage ranges of the selector switch *S*-5 overlap. By readjusting *R*-54, stabilized output higher than 2100 volts can be attained. The plate resistance of 100 megohms in the 6C6 amplifier tube, *V*-10, is the maximum that may be used. For higher values it was found that a large proportion of the 6C6 tubes exhibited internal leakage which upset stabilization.

The fuse indicated in Fig. 1 between the transformer winding and the high voltage rectifier tube (*V*-8) is necessary to protect the transformer in the event that the rectifier tube becomes gaseous and draws excessive current. A $\frac{1}{16}$-ampere fuse may be used but a $\frac{1}{32}$-ampere fuse is better insurance.

11. REFERENCE VOLTAGE FOR HIGH VOLTAGE STABILIZATION

The cathode of the 6C6 tube is connected to the low voltage supply through a two stage stabilizer circuit which provides the very constant reference voltage required. The first stage of this stabilization is supplied by the two voltage regulator tubes (*V*-13, *V*-14) and the second stage by two neon bulbs (*V*-11, *V*-12).

The use of neon bulbs for providing a constant reference voltage in place of batteries is common in many vacuum tube stabilizers. In order to get minimum dependence on input voltage, great care must be taken to operate the neon bulb in its most favorable current range, and often a number of bulbs must be tried to find one which is stable and at the same time gives the required constancy of voltage through a sufficient current range. An attempt to substitute *VR* type regulator tubes for neon bulbs showed that, while the output voltage (*ca.* 105 volts) of these tubes is independent of current over a much wider range than in the case of neon bulbs, there is an instability of 1 to 2 volts which makes them unsatisfactory for use as reference voltage in a high voltage stabilizer. This instability is caused by the fact that the "active" area of the cathode may vary at random with constant current, and this affects the voltage slightly. On the other hand, if a neon bulb carries enough current to activate the entire cathode, it is very stable (*ca.* 55 volts) at constant current, since the active area cannot vary. However, when operated in this fashion the rate of change of voltage with changing current is too great to allow its use in the conventional manner.

In order to get the excellent regulation against input voltage change provided by the *VR* type tubes and at the same time avoid the small fluctuations which they introduce, the circuit shown was developed. In this circuit the input voltage to the neon bulbs is made constant to within 1 or 2 volts against very large changes in the power supply voltage by the *VR* type tubes. After this preliminary stabilization, the neon bulbs, supplied through a resistor (*R*-40), remove the small random variations introduced by the *VR* type tubes, without introducing further fluctuations, since they are run with their entire cathodes "active." It should be noted that the neon bulbs require aging of about two weeks before they become constant in voltage.

12. ADJUSTMENT AND CALIBRATION

The following adjustments must be made when the counting rate meter is first put in operation:

(1) The high voltage adjusting potentiometer (*R*-54) should be adjusted for satisfactory control on all voltage ranges.

(2) The pulse equalizer bias (*R*-14) should be adjusted for maximum sensitivity, taking care to keep it well away from the point where the multivibrator has any tendency to go into oscillation. This is usually done by applying the 3600 pulses per minute and adjusting the multivibrator by ear until it is just below the point where a second harmonic can be heard.

(3) The vacuum tube voltmeter should be adjusted (*R*-31) to read zero when no signal is applied.

termining the error of the mean do not apply because of the interdependence of successive points on the output curve. With appropriate weighting factors to account for this interdependence it can be shown that[14]

$$r(T)=\frac{(1+2T/RC)^{\frac{1}{2}}}{(1+T/RC)}r_1. \tag{6}$$

Figure 3 is a plot of $r(T)/r_1$ as a function of the duration T/RC of the continuous observation. The ratio $r(T)/r_1$ is always less than $(2RC/T)^{\frac{1}{2}}$ because of the statistical information stored in the tank circuit before the beginning of the observation, while equilibrium was becoming established. Figure 3 shows that to reduce the probable error of the average value $r(T)$ to less than half of the probable error of a single observation r_1, the duration T of the observation must be $6.6RC$; while if $r(T)$ is to be a quarter of r_1 the observation must be $31RC$ long.

Thus to analyze an output record we (a) determine x by the "equal area" construction, (b) determine r_1 from the "width" of the fluctuations, and (c) determine $r(T)$ from r_1, T, and Fig. 3. The result of the measurement is then $x\pm r(T)$.

When an ink recording milliammeter is used for registering the output of the counting rate meter, the drag of the pen on the recording paper tends to suppress the largest fluctuations in the output current, especially since these usually have a short duration. In order to determine the quantitative effect of the pen-paper friction on estimates of r_1 we have compared the width of the fluctuations observed while simultaneously recording the output photographically from a critically damped 6-second galvanometer, and from a 5-ma. Esterline-Angus recording milliammeter. Tests were made at counting rates of 60, 150, 900, and 4500 counts per minute, using the ranges and tank-circuit time constants as shown in Table I, and with the galvanometer adjusted to give the same absolute value of the average deflection x as the ink recorder. The width of the photographic record is in good agreement with the theoretical width given by Eq. (5), and the ink record is 78 ± 8 percent of the theoretical width. Thus for ink recording we take the "half-width" of the record as $0.78\times 3r_1=2.3r_1$, instead of $3r_1$. But for visual observations from the panel meter, or for photographic recording, from a galvanometer whose period is much less than RC, we take the "half-width" of the fluctuations as $3r_1$.

As the counting rate meter can possess an absolute calibration in terms of counts per minute x, an independent estimate may be made of the probable error $r(T)$. This is analogous to the usual Poisson relation involving the total number of counts observed, but follows directly from Eq. (6), which reduces to

$$\frac{r(T)}{r_1}\leqslant(2RC/T)^{\frac{1}{2}}, \tag{7}$$

where the inequality refers to short observations, and the equality to $T\gg RC$. Combining with Eq. (5), we have:

$$r(T)\leqslant 0.6745(x/T)^{\frac{1}{2}}, \tag{8}$$

which approaches the Poisson value as $T\gg RC$.

16. STATISTICAL EQUILIBRIUM TIME

A finite time is required for the counting rate meter output to reach a new equilibrium value when the average counting rate is changed suddenly. We regard the new equilibrium as effectively reached when the theoretical instantaneous output current differs from the final average output by r_1 or less. Starting with zero

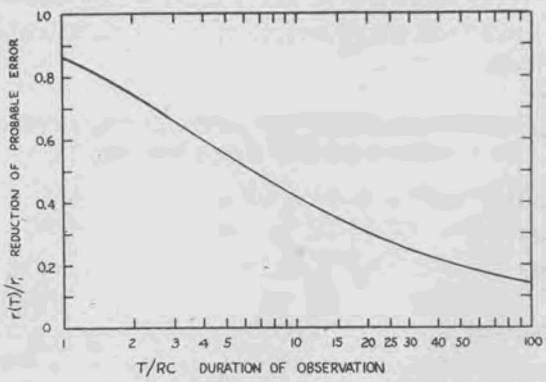

FIG. 3. Dependence Eq. (6) of the ratio of the probable error $r(T)$ of the average value x, to the probable error r_1 of an individual observation for continuous observations of various duration T/RC.

[14] Follows from Eq. (5) of reference 5.

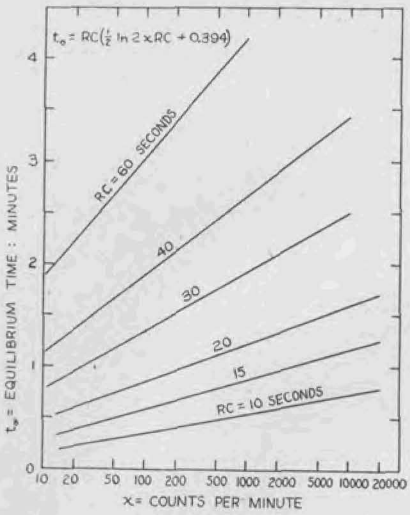

FIG. 4. Time t_0 required for the output to rise from zero to within r_1 of the final equilibrium value, for various counting rates x and time constants RC.

output current, this *equilibrium time* t_0 is[5]

$$t_0 = RC(\tfrac{1}{2} \ln 2xRC + 0.394). \tag{9}$$

Figure 4 shows plots of Eq. (9) for tank circuit time constants RC of 10, 20, 30, 40, and 60 seconds, and counting rates of 10 to 2×10^4 per minute. On the low count ranges (5, 6, 7) the 2 μf tank condenser of Fig. 1 involves an equilibrium time of several minutes. If desired, this can be more than halved by using only 1 μf, thus obtaining time constants which are half of those shown in Table I. As manual sample changing often requires a few minutes, the subsequent equilibrium time usually involves little relative delay even in routine measurements, especially since an experienced operator, listening to the counts in the loudspeaker monitor, can usually set the output nearly on its equilibrium value at the outset by using S-3 and S-1 to raise and lower the meter reading. Even with automatic sample changing equipment[11] the equilibrium time delay adds only slightly to the fraction of time elapsed between samples.

17. LINEARITY OF RESPONSE

The *linearity* of response of the counting rate meter, if supplied with infinitely short pulses from the preamplifier, would be determined by the largest time constant in the second amplifier and multivibrator stages. This is the relaxation time of the multivibrator which is about $\tau = 6\times 10^{-5}$ second. Pulses spaced more closely than τ would not be recorded, and the fraction of counts missed would be given by the interval law.[15] Then the ratio of true x_0 to observed x counting rate would be given closely by

$$x_0/x = 1 + x\tau. \tag{10}$$

At 20,000 counts per minute, $x\tau$ has the value 0.02, showing that only 2 percent of the counts would be missed. Thus the amplifier can be considered essentially linear up to its highest available counting speeds.

However, Geiger-Müller counters themselves may have a resolving time which is somewhat greater than 10^{-4} second. Significant variations from linearity may therefore be encountered at rates considerably below 20,000 per minute. The behavior of various types of counters and preamplifiers is often complex,[16] and an empirical linearity test should always be carried out on every counter used. This can be done most readily by measuring the γ-ray activity of a series of samples $a, b, c, \cdots$ and comparing these individual values with a direct measurement of $(a+b)$, $(a+b+c)$, etc. The γ-ray standards[17] distributed by the National Bureau of Standards are ideal for this purpose.

[15] L. I. Schiff, Phys. Rev. **50**, 88–96 (1936).
[16] A. Roberts, Rev. Sci. Inst. **12**, 71–76 (1941).
[17] L. F. Curtiss, *et al.*, Phys. Rev. **57**, 457L (1940).